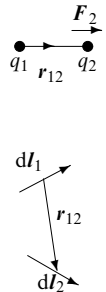


7.5 Force, torque, and energy

Electromagnetic force and torque

Force between two static charges: Coulomb's law	$\mathbf{F}_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2} \hat{\mathbf{r}}_{12} \quad (7.119)$	$\mathbf{F}_2$ force on q_2 $q_{1,2}$ charges $\mathbf{r}_{12}$ vector from 1 to 2 $\hat{\mathbf{r}}$ unit vector ϵ_0 permittivity of free space
Force between two current-carrying elements	$d\mathbf{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2} [d\mathbf{I}_2 \times (d\mathbf{I}_1 \times \hat{\mathbf{r}}_{12})] \quad (7.120)$	$d\mathbf{I}_{1,2}$ line elements $I_{1,2}$ currents flowing along $d\mathbf{I}_1$ and $d\mathbf{I}_2$ $d\mathbf{F}_2$ force on $d\mathbf{I}_2$ μ_0 permeability of free space
Force on a current-carrying element in a magnetic field	$d\mathbf{F} = I d\mathbf{l} \times \mathbf{B} \quad (7.121)$	$d\mathbf{l}$ line element $\mathbf{F}$ force I current flowing along $d\mathbf{l}$ $\mathbf{B}$ magnetic flux density
Force on a charge (Lorentz force)	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (7.122)$	$\mathbf{E}$ electric field $\mathbf{v}$ charge velocity
Force on an electric dipole ^a	$\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} \quad (7.123)$	$\mathbf{p}$ electric dipole moment
Force on a magnetic dipole ^b	$\mathbf{F} = (\mathbf{m} \cdot \nabla) \mathbf{B} \quad (7.124)$	$\mathbf{m}$ magnetic dipole moment
Torque on an electric dipole	$\mathbf{G} = \mathbf{p} \times \mathbf{E} \quad (7.125)$	$\mathbf{G}$ torque
Torque on a magnetic dipole	$\mathbf{G} = \mathbf{m} \times \mathbf{B} \quad (7.126)$	
Torque on a current loop	$\mathbf{G} = I_L \oint_{\text{loop}} \mathbf{r} \times (d\mathbf{I}_L \times \mathbf{B}) \quad (7.127)$	$d\mathbf{I}_L$ line-element (of loop) $\mathbf{r}$ position vector of $d\mathbf{I}_L$ I_L current around loop



^a $\mathbf{F}$ simplifies to $\nabla(\mathbf{p} \cdot \mathbf{E})$ if $\mathbf{p}$ is intrinsic, $\nabla(pE/2)$ if $\mathbf{p}$ is induced by $\mathbf{E}$ and the medium is isotropic.

^b $\mathbf{F}$ simplifies to $\nabla(\mathbf{m} \cdot \mathbf{B})$ if $\mathbf{m}$ is intrinsic, $\nabla(mB/2)$ if $\mathbf{m}$ is induced by $\mathbf{B}$ and the medium is isotropic.



Electromagnetic energy

Electromagnetic field energy density (in free space)	$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$	(7.128)	u energy density E electric field B magnetic flux density
Energy density in media	$u = \frac{1}{2}(\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H})$	(7.129)	ϵ_0 permittivity of free space μ_0 permeability of free space $\mathbf{D}$ electric displacement $\mathbf{H}$ magnetic field strength
Energy flow (Poynting) vector	$\mathbf{N} = \mathbf{E} \times \mathbf{H}$	(7.130)	c speed of light N energy flow rate per unit area $\perp$ to the flow direction
Mean flux density at a distance r from a short oscillating dipole	$\langle N \rangle = \frac{\omega^4 p_0^2 \sin^2 \theta}{32\pi^2 \epsilon_0 c^3 r^3} \mathbf{r}$	(7.131)	p_0 amplitude of dipole moment $\mathbf{r}$ vector from dipole ($\gg$ wavelength) θ angle between $\mathbf{p}$ and $\mathbf{r}$ ω oscillation frequency
Total mean power from oscillating dipole ^a	$W = \frac{\omega^4 p_0^2 / 2}{6\pi \epsilon_0 c^3}$	(7.132)	W total mean radiated power
Self-energy of a charge distribution	$U_{\text{tot}} = \frac{1}{2} \int_{\text{volume}} \phi(\mathbf{r}) \rho(\mathbf{r}) d\tau$	(7.133)	U_{tot} total energy $d\tau$ volume element $\mathbf{r}$ position vector of $d\tau$ ϕ electrical potential ρ charge density
Energy of an assembly of capacitors ^b	$U_{\text{tot}} = \frac{1}{2} \sum_i \sum_j C_{ij} V_i V_j$	(7.134)	V_i potential of i th capacitor C_{ij} mutual capacitance between capacitors i and j
Energy of an assembly of inductors ^c	$U_{\text{tot}} = \frac{1}{2} \sum_i \sum_j L_{ij} I_i I_j$	(7.135)	L_{ij} mutual inductance between inductors i and j
Intrinsic dipole in an electric field	$U_{\text{dip}} = -\mathbf{p} \cdot \mathbf{E}$	(7.136)	U_{dip} energy of dipole $\mathbf{p}$ electric dipole moment
Intrinsic dipole in a magnetic field	$U_{\text{dip}} = -\mathbf{m} \cdot \mathbf{B}$	(7.137)	$\mathbf{m}$ magnetic dipole moment
Hamiltonian of a charged particle in an EM field ^d	$H = \frac{ \mathbf{p}_m - q\mathbf{A} ^2}{2m} + q\phi$	(7.138)	H Hamiltonian $\mathbf{p}_m$ particle momentum q particle charge m particle mass $\mathbf{A}$ magnetic vector potential

^aSometimes called “Larmor’s formula.”

^b C_{ii} is the self-capacitance of the i th body. Note that $C_{ij} = C_{ji}$.

^c L_{ii} is the self-inductance of the i th body. Note that $L_{ij} = L_{ji}$.

^dNewtonian limit, i.e., velocity $\ll c$.

